Photoacoustic effect for multiply scattered light

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We consider the photoacoustic effect for multiply scattered light in a random medium. Within the accuracy of the diffusion approximation to the radiative transport equation, we present a general analysis of the sensitivity of a photoacoustic wave to the presence of one or more small absorbing objects. Applications to tumor detection by photoacoustic imaging are suggested.

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I. INTRODUCTION

The photoacoustic effect refers to the generation of acoustic waves due to the interaction of light with an absorbing medium [1]. It is well known that the emitted acoustic waves carry information about the medium [2,3]. This principle has been exploited in a variety of biomedical imaging modalities which combine the spectroscopic sensitivity of optical methods with the spatial resolution of ultrasonic imaging [4]. Two forms of photoacoustic imaging are usually distinguished. Direct imaging employs a focused ultrasound detector for image formation. Tomographic imaging utilizes an unfocused ultrasound detector and an inverse scattering method is used to reconstruct images of the optical properties of the medium.

The theory of the photoacoustic effect begins with a model for the propagation of electromagnetic waves in an absorbing medium. The most general such model is based on the Maxwell equations for a dielectric with a complex-valued permittivity. Alternatively, a description based on the radiative transport equation (RTE) or the diffusion approximation (DA) to the RTE may be employed. Such an approach is valid for a random medium in which multiple scattering of light occurs. Upon absorption of light, the medium undergoes local heating and thermal expansion. Typically in a fluid, a linear relationship between mass density and temperature is assumed, although nonlinear models have also been considered [5]. The mass density and the velocity field in the fluid are further related by the Navier-Stokes equation, which can be linearized to describe the propagation of small-amplitude pressure waves.

The propagation of photoacoustic waves is governed by the acoustic wave equation with a source term which is proportional to the time derivative of the intensity of the optical field. In an infinite nonscattering medium containing a spherical absorber, the pressure may be obtained explicitly [6–9]. The solution to this model problem is of fundamental interest and may be used to estimate the resolution of a photoacoustic imaging experiment in a transparent medium. However, the analysis must be modified if multiple scattering of light is to be accounted for. In this paper, we consider the related problem of the photoacoustic effect in a random medium containing one or more small absorbing objects. The propagation of light is described by the DA to the RTE. The calculations we report are exact and account for effects due to reflection of the photoacoustic wave at the boundary of the medium. We apply our results to the problem of estimating the minimum detectable size of a breast tumor in photoacoustic imaging.

The remainder of this paper is organized as follows. In Sec. II, the equations governing the photoacoustic effect in a random medium are derived. Then, in Sec. III, the point-absorber model is described. In Sec. IV, the theory of the photoacoustic effect for a small absorbing object is considered for the cases of infinite and semi-infinite media. Finally, the theory for a collection of absorbing objects is presented in Sec. V. Our conclusions are formulated in Sec. VI.

II. PHOTOACOUSTIC EFFECT

In this section we derive the basic equations governing the photoacoustic effect in a random medium. We begin by considering the propagation of light in a volume Ω which contains a fluid consisting of a suspension of scattering particles. The specific intensity $I(\mathbf{r},\hat{\mathbf{s}},t)$ at the point $\mathbf{r}\in\Omega$ in the direction $\hat{\mathbf{s}}$ at time t is assumed to obey the RTE [10],

$$\frac{1}{c}\frac{\partial I}{\partial t} + \hat{\mathbf{s}}\cdot\nabla I + \mu_a I - LI = S, \tag{1}$$

where

$$LI(\mathbf{r},\hat{\mathbf{s}},t) = \mu_s \int [f(\hat{\mathbf{s}}',\hat{\mathbf{s}})I(\mathbf{r},\hat{\mathbf{s}}',t) - f(\hat{\mathbf{s}},\hat{\mathbf{s}}')I(\mathbf{r},\hat{\mathbf{s}},t)]d^2s' . \tag{2}$$

Here c denotes the speed of light in the medium, μ_a and μ_s are the absorption and scattering coefficients, respectively, and S is the power density of the source. In general, we will allow μ_a to be position dependent and will assume that μ_s is constant throughout Ω. The phase function $f(\hat{\mathbf{s}},\hat{\mathbf{s}}')$ is normalized so that $\int f(\hat{\mathbf{s}},\hat{\mathbf{s}}')d^2s'=1$ for all $\hat{\mathbf{s}}$ and is assumed to depend only upon the angle between $\hat{\mathbf{s}}$ and $\hat{\mathbf{s}}'$, corresponding to scattering by spherically symmetric particles. The specific intensity satisfies a boundary condition of the form

$$I(\mathbf{r},\hat{\mathbf{s}}) = 0, \quad \hat{\mathbf{n}}\cdot\hat{\mathbf{s}} < 0, \quad \mathbf{r}\in\partial\Omega, \tag{3}$$

where $\hat{\mathbf{n}}$ is the outward unit normal to $\partial\Omega$. Thus no light enters Ω except due to the source. We will also introduce the angularly averaged specific intensity defined by

$$\bar{I}(\mathbf{r},t) = \frac{1}{4\pi}\int I(\mathbf{r},\hat{\mathbf{s}},t)d^2s. \tag{4}$$

Absorption of light leads to heat transfer by thermal diffusion. The temperature T obeys the diffusion equation

$$\rho C_p \frac{\partial T}{\partial t} - \kappa \nabla^2 T = \mu_a \bar{I}, \tag{5}$$

where ρ is the mass density, C_p is the constant-pressure heat capacity per unit mass, and κ is the thermal conductivity. Upon heating, the medium undergoes thermal expansion with subsequent generation of a pressure wave. The mass density thus satisfies the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \rho\beta \frac{\partial T}{\partial t}, \tag{6}$$

where β is the coefficient of volume thermal expansion. The velocity field $\mathbf{v}$ is taken to obey the Navier-Stokes equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v}), \tag{7}$$

where p is the pressure, η is the shear viscosity, and ζ is the bulk viscosity.

We suppose that the light incident on the medium is produced by a pulsed source and that the duration of the pulse is short in comparison to the time scale of thermal diffusion. In this situation, we may neglect the spatial dependence of T and thus, using Eq. (5), we see that Eq. (6) becomes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \frac{\beta}{C_p}\mu_a \bar{I}. \tag{8}$$

We can now derive the equation obeyed by small-amplitude photoacoustic waves. First, we note that if the source power is sufficiently small [11], we can write the density and pressure in the form

$$p = p_0 + \delta p, \tag{9}$$

$$\rho = \rho_0 + \delta\rho, \tag{10}$$

where p_0, ρ_0 are the equilibrium density and pressure before the arrival of the pulse and $\delta p, \delta\rho$ are fluctuations in the density and pressure with $\delta p \ll p_0$ and $\delta\rho \ll \rho_0$. Next, we linearize Eqs. (7) and (8) about constant p_0 and ρ_0,

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla \delta p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v}), \tag{11}$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \nabla\cdot\mathbf{v} = \frac{\beta\mu_a}{C_p}\bar{I}. \tag{12}$$

Using the above result, it is possible to eliminate the velocity field and obtain a single equation involving only δp and $\delta\rho$,

$$\frac{\partial^2 \delta\rho}{\partial t^2} = \nabla^2 \delta p + \left(\zeta + \frac{4}{3}\eta\right)\left(\frac{1}{\rho_0}\frac{\partial}{\partial t}\nabla^2\delta\rho - \frac{\beta\mu_a}{\rho_0 C_p}\nabla^2 \bar{I}\right) + \frac{\beta\mu_a}{C_p}\frac{\partial \bar{I}}{\partial t}. \tag{13}$$

Finally, we use the relation $\delta p = c_s^2 \delta\rho$, where c_s is the speed of sound and obtain the wave equation for the pressure

$$\frac{1}{c_s^2}\frac{\partial^2 p}{\partial t^2} = \nabla^2 p + \frac{\zeta + \frac{4}{3}\eta}{\rho_0 c_s^2}\left(\frac{\partial}{\partial t}\nabla^2 p - \Gamma\mu_a \nabla^2 \bar{I}\right) + \frac{\Gamma\mu_a}{c_s^2}\frac{\partial \bar{I}}{\partial t}. \tag{14}$$

Here we have introduced the Gruneisen constant $\Gamma = \beta c_s^2 / C_p$, which is a measure of the efficiency of the conversion of heat to pressure. Note that $\Gamma \approx 0.1$ in water at room temperature.

For simplicity, we restrict our attention to the case of time-harmonic fields with a $e^{-i\omega t}$ time dependence. The general case can be handled by Fourier superposition. The time-independent pressure then satisfies the reduced wave equation

$$\left(1 + i\frac{\omega}{\omega_0}\right)\nabla^2 p + k_0^2 p = \frac{i\omega\Gamma\mu_a}{c_s^2}\bar{I} - \frac{\Gamma\mu_a}{\omega_0}\nabla^2\bar{I}, \tag{15}$$

where the wave number $k_0 = \omega / c_s$ and

$$\omega_0 = \frac{\rho_0 c_s^2}{\zeta + \frac{4}{3}\eta}. \tag{16}$$

The frequency ω_0 determines the scale over which viscous attenuation of the pressure wave occurs. Since $\omega \ll \omega_0$ for most cases of practical interest, we will neglect effects due to viscosity (this point is discussed further in Sec. IV B) and thus Eq. (15) becomes

$$\nabla^2 p + k_0^2 p = \frac{i\omega\Gamma\mu_a}{c_s^2}\bar{I}. \tag{17}$$

The solution to Eq. (17) is given by

$$p(\mathbf{r}) = \frac{i\omega\Gamma}{c_s^2}\int d^3r' G(\mathbf{r},\mathbf{r}')\bar{I}(\mathbf{r}')\mu_a(\mathbf{r}'), \tag{18}$$

where the speed of sound is assumed to be constant and the Green's function G obeys the equation

$$\nabla^2 G(\mathbf{r},\mathbf{r}') + k_0^2 G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'). \tag{19}$$

The Green's function also obeys appropriate boundary conditions.

If the incident optical field is generated by a point source at $\mathbf{r}_1$ pointing in the direction $\hat{\mathbf{s}}_1$, then the angularly averaged specific intensity is given by

$$\bar{I}(\mathbf{r}) = \frac{S_0}{4\pi}\int d^2s G(\mathbf{r},\hat{\mathbf{s}};\ \mathbf{r}_1,\hat{\mathbf{s}}_1), \tag{20}$$

where S_0 is the source power. Here G is the Green's function for the time-independent RTE,

$$\hat{\mathbf{s}}\cdot\nabla G(\mathbf{r},\hat{\mathbf{s}};\ \mathbf{r}',\hat{\mathbf{s}}') + (\mu_a - i\omega/c)G(\mathbf{r},\hat{\mathbf{s}};\ \mathbf{r}',\hat{\mathbf{s}}') - LG(\mathbf{r},\hat{\mathbf{s}};\ \mathbf{r}',\hat{\mathbf{s}}') = \delta(\mathbf{r}-\mathbf{r}')\delta(\hat{\mathbf{s}}-\hat{\mathbf{s}}'). \tag{21}$$

Combining Eqs.(18) and (20), we find that the pressure is given by

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_s^2}\int d^3r' d^2s' G(\mathbf{r},\mathbf{r}')G(\mathbf{r}',\hat{\mathbf{s}}';\ \mathbf{r}_1,\hat{\mathbf{s}}_1)\mu_a(\mathbf{r}'). \tag{22}$$

Equation (22) is a general expression for the pressure of a photoacoustic wave. An important special case is obtained when the DA to the RTE applies. Following [12], the DA is obtained by expanding the Green's function G in angular harmonics. To lowest order, it can be seen that

$$G(\mathbf{r},\hat{\mathbf{s}};\mathbf{r},\hat{\mathbf{s}}') = \frac{c}{4\pi}(1 + \ell^*\hat{\mathbf{s}} \cdot \nabla_{\mathbf{r}})(1 - \ell^*\hat{\mathbf{s}}' \cdot \nabla_{\mathbf{r}'})G_D(\mathbf{r},\mathbf{r}'), \tag{23}$$

where the transport mean free path $\ell^* = 1/(\mu_a + \mu_s')$ with $\mu_s' = (1-g)\mu_s$, g being the anisotropy of the phase function f. The diffusion Green's function $G_D(\mathbf{r},\mathbf{r}')$ satisfies the equation

$$-D\nabla^2 G_D(\mathbf{r},\mathbf{r}') + (\alpha - i\omega)G_D(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{24}$$

where the diffusion coefficient $D = 1/3c\ell^*$ and $\alpha = c\mu_a$. The diffusion Green's function also satisfies the boundary condition

$$G_D(\mathbf{r},\mathbf{r}') + \ell\hat{\mathbf{n}} \cdot \nabla G_D(\mathbf{r},\mathbf{r}') = 0, \quad \mathbf{r},\mathbf{r}' \in \partial\Omega, \tag{25}$$

where ℓ is the extrapolation length [10]. The DA is valid when $\ell^*|\nabla G_D| \ll G_D$ and breaks down in optically thin layers; in weakly scattering or strongly absorbing media, that is with $\mu_s \ll \mu_a$; and near boundaries. One or more of these conditions are often met in biomedical applications.

Within the accuracy of the DA, Eq. (22) becomes

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_s^2}\left(1 - \frac{\ell^*}{\ell}\right)\int d^3r' G(\mathbf{r},\mathbf{r}')G_D(\mathbf{r}',\mathbf{r}_1)\alpha(\mathbf{r}'), \tag{26}$$

where we have made use of the boundary condition Eq. (25) and have assumed that the source is oriented in the inward normal direction. In the case of an infinite medium, the diffusion Green's function does not obey the boundary condition Eq. (25), but instead vanishes at infinity. Thus the gradient terms in Eq. (23) are small and the term in parentheses on the right-hand side of Eq. (26) does not appear.

III. POINT-ABSORBER MODEL

In this section we consider the propagation of diffuse light in the presence of a small absorbing object. The goal is to compute the diffusion Green's function G_D. As shown in [12], G_D satisfies the integral equation

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \int d^3r'' G_D^{(0)}(\mathbf{r},\mathbf{r}'')\alpha_1(\mathbf{r}'')G_D(\mathbf{r}'',\mathbf{r}'). \tag{27}$$

Here $G_D^{(0)}$ is the Green's function for a homogeneous medium with absorption α_0 and $\alpha_1 = \alpha - \alpha_0$. The unperturbed Green's function $G_D^{(0)}$ satisfies Eq. (24) with $\alpha = \alpha_0$ and obeys the boundary condition Eq. (25). In an infinite medium, $G_D^{(0)}$ is given by the expression

$$G_D^{(0)}(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi D}\frac{e^{-k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \tag{28}$$

where the diffuse wave number is defined by $k = \sqrt{(\alpha_0 - i\omega)/D}$. In a semi-infinite medium, corresponding to the half-space $z \geq 0$, it can be seen [12] that $G_D^{(0)}$ can be expanded into two-dimensional plane waves,

$$G_D^{(0)}(\mathbf{r},\mathbf{r}') = \frac{1}{2D(2\pi)^2}\int \frac{d^2q}{Q(q)} e^{i\mathbf{q}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')} \times\left(e^{-Q(q)|z-z'|} + \frac{Q(q)\ell - 1}{Q(q)\ell + 1}e^{-Q(q)|z+z'|}\right), \tag{29}$$

where we have used the notation $\mathbf{r} = (\boldsymbol{\rho}, z)$ and $Q(q) = \sqrt{q^2 + k^2}$.

Consider an absorbing object whose size is small compared to the decay length $1/k$. Suppose that the object is embedded in a homogeneous medium with absorption α_0. Then the total absorption of the medium is taken to be

$$\alpha(\mathbf{r}) = \alpha_0 + \delta\alpha_0 V\delta(\mathbf{r}-\mathbf{r}_0), \tag{30}$$

where $\mathbf{r}_0$ denotes the position of the absorber, $\delta\alpha_0$ is its absorption, and V is its volume. Equation (30) defines the point-absorber model. Inserting Eq. (30) into the integral equation (27), we find that the diffusion Green's function satisfies an algebraic equation of the form

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \delta\alpha_0 V G_D^{(0)}(\mathbf{r},\mathbf{r}_0)G_D(\mathbf{r}_0,\mathbf{r}'). \tag{31}$$

Equation (31) can be rewritten in the form of the geometric series

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \delta\alpha_0 V G_D^{(0)}(\mathbf{r},\mathbf{r}_0)G_D^{(0)}(\mathbf{r}_0,\mathbf{r}') + (\delta\alpha_0 V)^2 G_D^{(0)}(\mathbf{r},\mathbf{r}_0)G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)G_D^{(0)}(\mathbf{r}_0,\mathbf{r}') + \cdots, \tag{32}$$

which can be summed with the result

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \delta\alpha V G_D^{(0)}(\mathbf{r},\mathbf{r}_0)G_D^{(0)}(\mathbf{r}_0,\mathbf{r}'). \tag{33}$$

Here the renormalized absorption $\delta\alpha$ is defined by the expression

$$\delta\alpha = \frac{\delta\alpha_0}{1 + \delta\alpha_0 V G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)}. \tag{34}$$

Equation (33) is an exact result for the Green's function of the point-absorber model.

The physical interpretation of the renormalized absorption requires some care. The quantity $G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)$ is divergent and thus $\delta\alpha$ vanishes. In order to remedy the situation, it is necessary to regularize the divergence. For the case of an infinite medium, we examine the behavior of $G_D^{(0)}(\mathbf{r},\mathbf{r}')$ for small $|\mathbf{r}-\mathbf{r}'|$,

$$G_D^{(0)}(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi D|\mathbf{r}-\mathbf{r}'|} - \frac{k}{4\pi D} + O(|\mathbf{r}-\mathbf{r}'|). \quad (35)$$

It can be seen that the singular part of $G_D^{(0)}$ is isolated in the first term above. Now, consider the Fourier integral representation

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{k^2}. \quad (36)$$

We introduce a high-frequency cutoff on the wave-vector integration to regularize the divergence,

$$G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0) = \frac{1}{4\pi D}\int_{|\mathbf{k}|\le(2\pi/\Lambda)} \frac{d^3k}{(2\pi)^3}\frac{1}{k^2} - \frac{k}{4\pi D}$$
$$= \frac{1}{4\pi D}\left(\frac{1}{\pi\Lambda} - k\right), \quad (37)$$

where Λ defines the cutoff. We may identify Λ with the linear size of the absorber and thus $k\Lambda \ll 1$. Note that $G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)$ does not depend upon $\mathbf{r}_0$, as may be expected from translational invariance.

For the case of a semi-infinite medium, we must also regularize $G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)$. Here, we note that the first term of Eq. (29) corresponds to $G_D^{(0)}$ for an infinite medium. Thus,

$$G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0) = \frac{1}{4\pi D}\left(\frac{1}{\pi\Lambda} - k\right) + \frac{1}{4\pi D}\int_0^\infty dq \frac{q}{Q(q)}\frac{Q(q)\ell - 1}{Q(q)\ell + 1}e^{-2Q(q)z_0}$$
$$= \frac{1}{4\pi D}\left(\frac{1}{\pi\Lambda} - k + \frac{1}{2z_0}e^{-2kz_0} - \frac{2}{\ell}E_1[2(k+1/\ell)z_0]\right), \quad (38)$$

where E_1 denotes the exponential integral defined by

$$E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt. \quad (39)$$

Using the asymptotic expansion

$$E_1(z) \sim \frac{e^{-z}}{z}\left[1 - \frac{1}{z} + O\left(\frac{1}{z^2}\right)\right], \quad (40)$$

we obtain

$$G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0) = \frac{1}{4\pi D}\left[\frac{1}{\pi\Lambda} - k + \frac{1}{z_0}e^{-2kz_0}\left(\frac{1}{2} - \frac{1}{k\ell}\right)\right] + O\left(\frac{1}{z_0^2}\right). \quad (41)$$

In contrast to the case of an infinite medium, $G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)$ depends upon z_0, as follows from the broken translational invariance in the z direction.

IV. PHOTOACOUSTIC EFFECT WITH A SMALL ABSORBER

In this section we consider the photoacoustic effect in the context of the point-absorber model. We treat separately the cases of infinite and semi-infinite media. We assume that the speed of sound is constant everywhere in space or in each half-space, respectively.

A. Infinite medium

We consider an infinite homogeneous medium with acoustic wave number k_0. The Green's function for the wave equation (17), which behaves as an outgoing wave, is given by

$$G(\mathbf{r},\mathbf{r}') = -\frac{1}{4\pi}\frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \quad (42)$$

Using Eq. (26), we see that the pressure due to the presence of a small absorber is given by the expression

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_s^2}\int d^3r' G(\mathbf{r},\mathbf{r}')G_D(\mathbf{r}',\mathbf{r}_1)\alpha(\mathbf{r}'), \quad (43)$$

where α is defined by Eq. (30) and G_D is given by Eqs. (33) and (28). The integral is readily evaluated and consists of a sum of four terms,

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_s^2}[p_1(\mathbf{r}) + p_2(\mathbf{r}) + p_3(\mathbf{r}) + p_4(\mathbf{r})]. \quad (44)$$

Here

$$p_1(\mathbf{r}) = -\frac{1}{4\pi}\frac{\alpha_0}{Dk_0^2 + \alpha_0 - i\omega}\left(\frac{e^{ik_0|\mathbf{r}-\mathbf{r}_1|}}{|\mathbf{r}-\mathbf{r}_1|} - \frac{e^{-k|\mathbf{r}-\mathbf{r}_1|}}{|\mathbf{r}-\mathbf{r}_1|}\right), \quad (45)$$

$$p_2(\mathbf{r}) = -\frac{\delta\alpha_0 V}{(4\pi)^2 D}\frac{e^{ik_0|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\frac{e^{-k|\mathbf{r}_0-\mathbf{r}_1|}}{|\mathbf{r}_0-\mathbf{r}_1|}, \quad (46)$$

$$p_3(\mathbf{r}) = \frac{\alpha_0\delta\alpha V}{4\pi D(Dk_0^2 + \alpha_0 - i\omega)}\frac{e^{-k|\mathbf{r}_1-\mathbf{r}_0|}}{|\mathbf{r}_1-\mathbf{r}_0|}\left(\frac{e^{ik_0|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|} - \frac{e^{-k|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\right), \quad (47)$$

$$p_4(\mathbf{r}) = \frac{\delta\alpha_0\delta\alpha V^2}{(4\pi)^3 D^2}\frac{e^{-k|\mathbf{r}_1-\mathbf{r}_0|}}{|\mathbf{r}_1-\mathbf{r}_0|}\frac{e^{ik_0|\mathbf{r}-\mathbf{r}_0|}}{|\mathbf{r}-\mathbf{r}_0|}\left(\frac{1}{\pi\Lambda} - k\right). \quad (48)$$

Evidently, p_1 corresponds to the pressure for a homogeneous medium with absorption α_0; the remaining three terms result from the presence of the absorbing object.

It is instructive to consider the asymptotic behavior of p_1 in the far zone. It can be seen that the far-field pressure behaves as an outgoing spherical wave of the form

$$p_1 \sim \frac{e^{ik_0 r}}{r}A, \quad (49)$$

where A is defined by

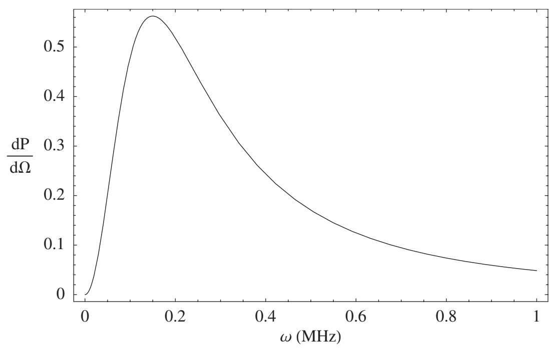

FIG. 1. Plot of the angular distribution of radiated power (in arbitrary units) as a function of frequency.

$$A = -\frac{i\omega\Gamma S_0}{(4\pi c_s)^2}\frac{\alpha_0}{Dk_0^2+\alpha_0-i\omega}e^{-ik_0\hat{\mathbf{r}}\cdot\mathbf{r}_1}. \tag{50}$$

Using this result, we find that the angular distribution of radiated power is given by the expression

$$\frac{dP}{d\Omega} = \frac{1}{2\rho_0 c_s}|A|^2 = \frac{\alpha_0^2\Gamma^2 S_0^2}{2\rho_0 c_s(4\pi c_s)^4}\frac{\omega^2}{(Dk_0^2+\alpha_0^2)^2+\omega^2}. \tag{51}$$

We note that the radiated power is isotropic, vanishes at $\omega=0$ and has a maximum at $\omega_{\max}=c_s\sqrt{\alpha_0/D}$. We further note that $dP/d\Omega \sim 1/\omega^2$ for large ω, consistent with the expected breakdown of the DA at high frequencies.

We now estimate the sensitivity of the photoacoustic pressure to the presence of a small absorbing object and the corresponding image resolution. We work in the geometry in which the source and the absorber are collinear. The speed of sound is $c_s=1.5\times10^5$ cm s^{-1}, the background absorption $\alpha_0=1$ ns^{-1}, and the diffusion coefficient $D=1$ cm^2 ns^{-1}. This choice of parameters is typical for breast tissue in the near infrared. Figure 1 illustrates the frequency dependence of $dP/d\Omega$ which shows a maximum at $\omega_{\max}\approx200$ kHz. At this frequency, the resolution is quite low, as can be seen by examining the quantity $|p|$. In Fig. 2, $|p|$ is plotted along the line $y=0$ in the $z=0$ plane when the source is located at the origin, the absorber is located at a depth $z_0=5$ cm, the contrast $\delta\alpha_0/\alpha_0=3$, and the linear size of the absorber is Λ $=0.5$ cm. The resolution can be identified with the full width at half-maximum (FWHM) of this curve. At a frequency of $\omega/2\pi=200$ kHz the FHWM$\approx$1 cm, while at $\omega/2\pi=1$ MHz the FWHM$\approx$0.2 cm. Evidently, at higher frequencies the resolution increases, as may be expected. Next, we define the relative change in pressure due to the object

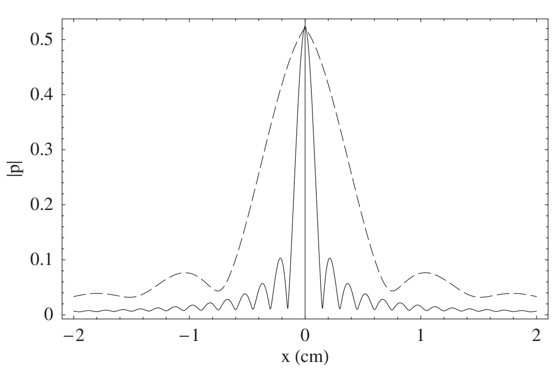

FIG. 2. $|p|$ (in arbitrary units) as a function of x in an infinite medium for different frequencies $\omega/2\pi=200$ kHz (dashed line) and $\omega/2\pi=1$ MHz (solid line).

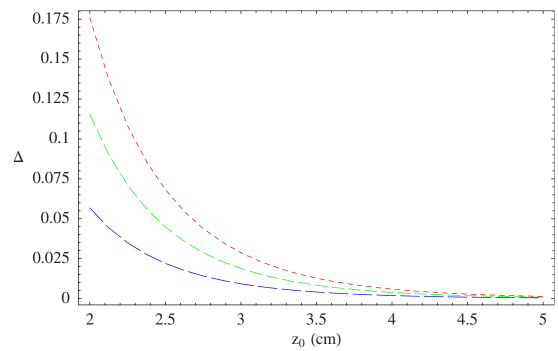

FIG. 3. (Color online) Δ as a function of z_0 in an infinite medium for different values of the contrast $\delta\alpha_0/\alpha_0=1$ (— — —), 2 (– – –), and 3 (- - -).

$$\Delta = \frac{|p-p_1|}{|p_1|}. \tag{52}$$

Note that this quantity is directly measurable in an experiment which utilizes a piezoelectric ultrasound transducer [13]. In Fig. 3, the parameter Δ is plotted as a function of the distance z_0 of the absorber from the source and a collinear detector for different values of the absorption contrast $\delta\alpha_0/\alpha_0$ at the frequency $\omega/2\pi=1$ MHz. The quantity Δ can be interpreted as the precision with which the pressure p can be measured relative to the background pressure p_1. For a fixed value of Δ, we can then estimate the threshold for the detection of the absorbing object; if Δ exceeds the experimental noise level we will say that an object is detectable. Note that we have not considered the absolute sensitivity of any particular instrument; this is beyond the scope of the present work. For example, if we set $\Delta=.01$, then it is possible to detect the object at a depth of 4 cm with a contrast $\delta\alpha_0/\alpha_0=3$. We note that this level of contrast is reasonable for a breast tumor. At lower contrast, the depth at which the object can be detected decreases, while at higher contrast, the depth increases.

B. Semi-infinite medium

We consider a planar interface between two homogeneous media. Medium 1 corresponds to the half-space $z<0$ and has acoustic wave number k_1. Medium 2 is the half-space $z>0$. It is filled with a highly scattering medium with acoustic wave number k_2 and contains a small absorbing object. In medium 1 the Green's function for the wave equation (17) obeys the equation

$$\nabla^2 G_1(\mathbf{r},\mathbf{r}') + k_1^2 G_1(\mathbf{r},\mathbf{r}') = 0, \tag{53}$$

while in medium 2 it obeys

$$\nabla^2 G_2(\mathbf{r},\mathbf{r}') + k_2^2 G_2(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{54}$$

where $z' \geq 0$ and $z \leq 0$. In addition, the Green's function must obey the boundary conditions

$$G_1|_{z=0} = G_2|_{z=0}, \tag{55}$$

$$\frac{1}{\rho_1}\left.\frac{\partial G_1}{\partial z}\right|_{z=0} = \frac{1}{\rho_2}\left.\frac{\partial G_2}{\partial z}\right|_{z=0}, \tag{56}$$

where ρ_1 and ρ_2 denote the mass densities in medium 1 and medium 2, respectively. By expanding G_1 and G_2 into plane-wave modes, it can be seen that

$$G_1(\mathbf{r},\mathbf{r}') = -\frac{i}{2(2\pi)^2}\int \frac{d^2q}{k_{2z}(q)} T(q)\exp[i\mathbf{q}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}') + ik_{2z}(q)z' - ik_{1z}(q)z] \tag{57}$$

and

$$G_2(\mathbf{r},\mathbf{r}') = -\frac{i}{2(2\pi)^2}\int \frac{d^2q}{k_{2z}(q)} e^{i\mathbf{q}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}[e^{ik_{2z}(q)|z-z'|} + R(q)e^{ik_{2z}(q)(z+z')}]. \tag{58}$$

Here R and T are the reflection and transmission coefficients which are given by the expressions

$$R(q) = \frac{k_{2z}(q) - \rho_{21}k_{1z}(q)}{k_{2z}(q) + \rho_{21}k_{1z}(q)}, \tag{59}$$

$$T(q) = \frac{2k_{2z}(q)}{k_{2z}(q) + \rho_{21}k_{1z}(q)}, \tag{60}$$

where $\rho_{21} = \rho_2/\rho_1$ and $k_{nz}(q) = \sqrt{k_n^2 - q^2}$ for $n=1,2$.

We can now calculate the acoustic pressure due to the presence of a small absorber. We assume that the absorber is located in medium 2 and that the source and detector are placed on the $z=0$ plane with coordinates $\mathbf{r}=(\boldsymbol{\rho},0)$ and $\mathbf{r}_1 = (\boldsymbol{\rho}_1,0)$. To proceed, we make use of Eq. (26) which becomes

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_{s2}^2}\left(1-\frac{\ell^*}{\ell}\right)\int d^3r' G_1(\mathbf{r},\mathbf{r}')G_D(\mathbf{r}',\mathbf{r}_1)\alpha(\mathbf{r}'), \tag{61}$$

where α is defined by Eq. (30) and G_D is given by Eqs.(33) and (38). Carrying out the integral, we find that p can be expressed as a sum of four terms,

$$p(\mathbf{r}) = \frac{i\omega\Gamma S_0}{4\pi c_{s2}^2}\left(1-\frac{\ell^*}{\ell}\right)[p_1(\mathbf{r}) + p_2(\mathbf{r}) + p_3(\mathbf{r}) + p_4(\mathbf{r})], \tag{62}$$

where

$$p_1(\mathbf{r}) = -\frac{i\alpha_0\ell}{4\pi D}\int_0^\infty \frac{dq}{k_{2z}(q)}\frac{qT(q)J_0(q|\boldsymbol{\rho}-\boldsymbol{\rho}_1|)}{[Q(q)-ik_{2z}(q)][Q(q)\ell+1]}, \tag{63}$$

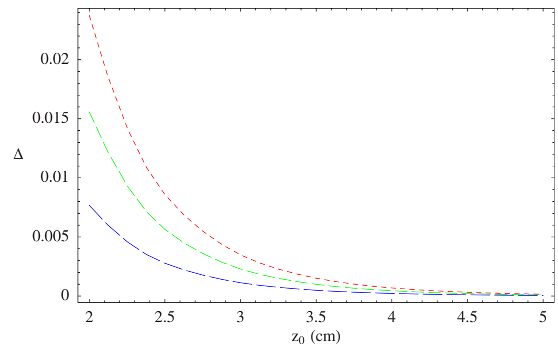

FIG. 4. (Color online) Δ as a function of z_0 in a semi-infinite medium for different values of the contrast $\delta\alpha_0/\alpha_0 = 1$ (— — —), 2 (– – –), and 3 (- - -).

$$p_2(\mathbf{r}) = \frac{\alpha_0\delta\alpha V}{D(k^2+k_2^2)}G_D^{(0)}(\mathbf{r}_1,\mathbf{r}_0)\left(G_1(\mathbf{r},\mathbf{r}_0) + \frac{D}{2}G_D^{(0)}(\mathbf{r},\mathbf{r}_0) - \frac{i}{8\pi}\int_0^\infty \frac{dq}{k_{2z}(q)} qT(q)J_0(q|\boldsymbol{\rho}-\boldsymbol{\rho}_1|)\frac{Q(q)\ell-1}{Q(q)\ell+1}e^{-Q(q)z_0}\right), \tag{64}$$

$$p_3(\mathbf{r}) = \delta\alpha_0 V G_D^{(0)}(\mathbf{r}_1,\mathbf{r}_0)G_1(\mathbf{r}_0,\mathbf{r}), \tag{65}$$

$$p_4(\mathbf{r}) = -\delta\alpha_0\delta\alpha V^2 G_D^{(0)}(\mathbf{r}_1,\mathbf{r}_0)G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)G_1(\mathbf{r}_0,\mathbf{r}). \tag{66}$$

We note that in the above formula, p_1 corresponds to the pressure that would be observed in the absence of the absorber. We also note that the term $G_D^{(0)}(\mathbf{r}_0,\mathbf{r}_0)$, which appears in the expression for p_4, must be regularized as in Eq. (38).

We now use the above result to estimate the detection threshold for a small absorbing inhomogeneity. As before, we consider the geometry in which the source and detector positions coincide and are collinear with the point absorber. Medium 1 is taken to have the acoustic properties of air at STP with speed of sound $c_{s1} = 3.4\times10^4$ cm s^{-1} and density $\rho_1 = 1.2\times10^{-3}$ g cm^{-3}. To account for absorption of the acoustic wave, a small imaginary part is added to the wave number k_1,

$$k_1 = \frac{\omega}{c_{s1}}\left(1 + i\frac{\omega}{2\omega_0}\right). \tag{67}$$

Note that in air $\omega_0 = 3\times10^8$ s^{-1}. In medium 2 the speed of sound is taken to be $c_{s2} = 1.5\times10^5$ cm s^{-1}, the background absorption $\alpha_0 = 1$ ns^{-1}, the diffusion coefficient $D = 1$ cm^2 ns^{-1}, the density $\rho_2 = 1$ g cm^{-3}, and the extrapolation length $\ell = 0.1$ cm. The attenuation of the pressure wave in water at a frequency of 1 MHz is extremely small ($\omega_0 \approx 10^{12}$ s^{-1}) and will be neglected.

Figure 4 shows a plot of Δ as a function of the distance z_0 for different values of the absorption contrast $\delta\alpha_0/\alpha_0$. The absorber is taken to have linear size $\Lambda = 0.5$ cm. We see that for a noise level $\Delta = .01$, it is possible to detect the object at a depth of 2.5 cm with a contrast $\delta\alpha_0/\alpha_0 = 3$. We note that this depth is smaller than the corresponding estimate in the case of an infinite medium, as may be expected due to absorption of light and ultrasound at the boundary.

V. MULTIPLE ABSORBERS

In this section we generalize our previous results to the case of a collection of point absorbers. We begin by deriving the appropriate diffusion Green's function and then use this result to calculate the acoustic pressure.

A. Diffusion Green's function

We consider a homogeneous medium containing a collection of N point absorbers with positions $\mathbf{R}_1, \ldots, \mathbf{R}_N$, absorptions $\delta\alpha_1, \ldots, \delta\alpha_N$, and volumes $V_1, \ldots, V_N$. The total absorption of the medium is given by

$$\alpha(\mathbf{r}) = \alpha_0 + \sum_i \delta\alpha_i V_i \delta(\mathbf{r} - \mathbf{R}_i), \tag{68}$$

where α_0 is the background absorption. For convenience, each absorber is assumed to have a distinct volume, even though only the product of $\delta\alpha_i$ and V_i arises in Eq. (68). Inserting the above expression for α into the integral equation (27), we find that the diffusion Green's function obeys the relation

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \sum_i G_D^{(0)}(\mathbf{r},\mathbf{R}_i)\delta\alpha_i V_i G_D(\mathbf{R}_i,\mathbf{r}'). \tag{69}$$

Evidently, Eq. (69) determines G_D self-consistently. This observation leads to a system of algebraic equations for $G_D(\mathbf{R}_i,\mathbf{r})$,

$$\sum_j M_{ij} G_D(\mathbf{R}_j,\mathbf{r}) = G_D^{(0)}(\mathbf{R}_i,\mathbf{r}), \tag{70}$$

where

$$M_{ij} = \delta_{ij} + G_D^{(0)}(\mathbf{R}_i,\mathbf{R}_j)\delta\alpha_j V_j. \tag{71}$$

Solving Eq. (70) for $G_D(\mathbf{R}_i,\mathbf{r})$ we obtain

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \sum_{i,j} G_D^{(0)}(\mathbf{r},\mathbf{R}_i) T_{ij} G_D^{(0)}(\mathbf{R}_j,\mathbf{r}'), \tag{72}$$

where $T_{ij} = \delta\alpha_i V_i M_{ij}^{-1}$ is the analog of the renormalized absorption. Note that as in Eqs.(37) and (38), the diagonal elements of M need to be properly regularized. If the absorbers are well separated, such that $k|\mathbf{R}_i - \mathbf{R}_j| \gg 1$, then $M_{ij} = \delta_{ij}[1 + \delta\alpha_i V_i G_D^{(0)}(\mathbf{R}_i,\mathbf{R}_i)]$ and

$$G_D(\mathbf{r},\mathbf{r}') = G_D^{(0)}(\mathbf{r},\mathbf{r}') - \sum_i G_D^{(0)}(\mathbf{r},\mathbf{R}_i)\frac{\delta\alpha_i V_i}{1 + \delta\alpha_i V_i G_D^{(0)}(\mathbf{R}_i,\mathbf{R}_i)} \times G_D^{(0)}(\mathbf{R}_i,\mathbf{r}'). \tag{73}$$

In this case, G_D corresponds to the superposition of the Green's functions for N isolated point absorbers. However, if the absorbers are sufficiently close to interact, then M depends in a nontrivial way upon the positions of *all* the absorbers in the system.

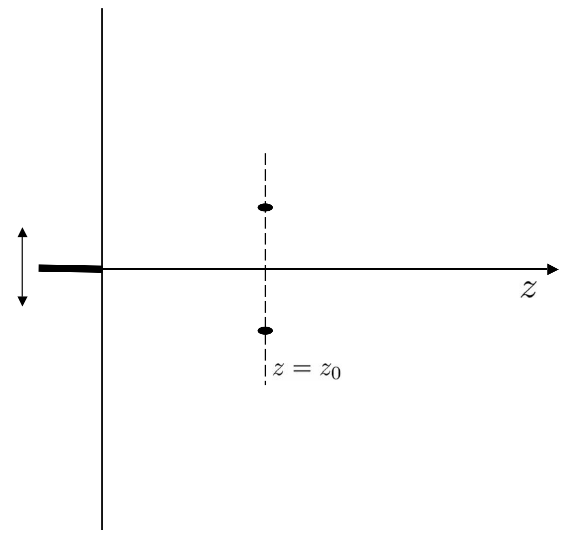

FIG. 5. Illustrating the geometry. The source and detector are scanned on the $z=0$ plane and the absorbers are located on the plane $z=z_0$.

B. Photoacoustic effect with multiple absorbers

We first consider the case of an infinite medium. The pressure is obtained by evaluating the integral Eq. (43) with G given by Eq. (42), G_D by Eq. (72) and α by Eq. (68). The result is of the form Eq. (44) with $p_1(\mathbf{r})$ given by Eq. (45) and

$$p_2(\mathbf{r}) = -\frac{1}{(4\pi)^2 D}\sum_i \delta\alpha_i V_i \frac{e^{ik_0|\mathbf{r}-\mathbf{R}_i|}}{|\mathbf{r}-\mathbf{R}_i|}\frac{e^{-k|\mathbf{r}_1-\mathbf{R}_i|}}{|\mathbf{r}_1-\mathbf{R}_i|}, \tag{74}$$

$$p_3(\mathbf{r}) = \frac{\alpha_0}{4\pi D(Dk_0^2 + \alpha_0 - i\omega)}\sum_{i,j} T_{ij}\frac{e^{-k|\mathbf{r}_1-\mathbf{R}_j|}}{|\mathbf{r}_1-\mathbf{R}_j|}\left(\frac{e^{ik_0|\mathbf{r}-\mathbf{R}_i|}}{|\mathbf{r}-\mathbf{R}_i|} - \frac{e^{-k|\mathbf{r}-\mathbf{R}_i|}}{|\mathbf{r}-\mathbf{R}_i|}\right), \tag{75}$$

$$p_4(\mathbf{r}) = \frac{1}{(4\pi)^3 D^2}\sum_{i,j,k}{}' \delta\alpha_k V_k T_{ij}\frac{e^{-k|\mathbf{r}_1-\mathbf{R}_j|}}{|\mathbf{r}_1-\mathbf{R}_j|}\frac{e^{ik_0|\mathbf{r}-\mathbf{R}_k|}}{|\mathbf{r}-\mathbf{R}_k|}\frac{e^{-k|\mathbf{R}_i-\mathbf{R}_k|}}{|\mathbf{R}_i-\mathbf{R}_k|} + \frac{1}{(4\pi)^3 D^2}\sum_{i,j}\delta\alpha_i V_i T_{ij}\frac{e^{-k|\mathbf{r}_1-\mathbf{R}_j|}}{|\mathbf{r}_1-\mathbf{R}_j|}\frac{e^{ik_0|\mathbf{r}-\mathbf{R}_i|}}{|\mathbf{r}-\mathbf{R}_i|}\left(\frac{1}{\pi\Lambda} - k\right), \tag{76}$$

where the prime on the summation excludes terms with $i = k$ and the index k is not to be confused with the wave number.

Next we consider the case of a semi-infinite medium. The pressure is obtained by evaluating the integral Eq. (61) with G_D given by Eq. (72) and α by Eq. (68). The result is of the form Eq. (62) with $p_1(\mathbf{r})$ given by Eq. (63) and

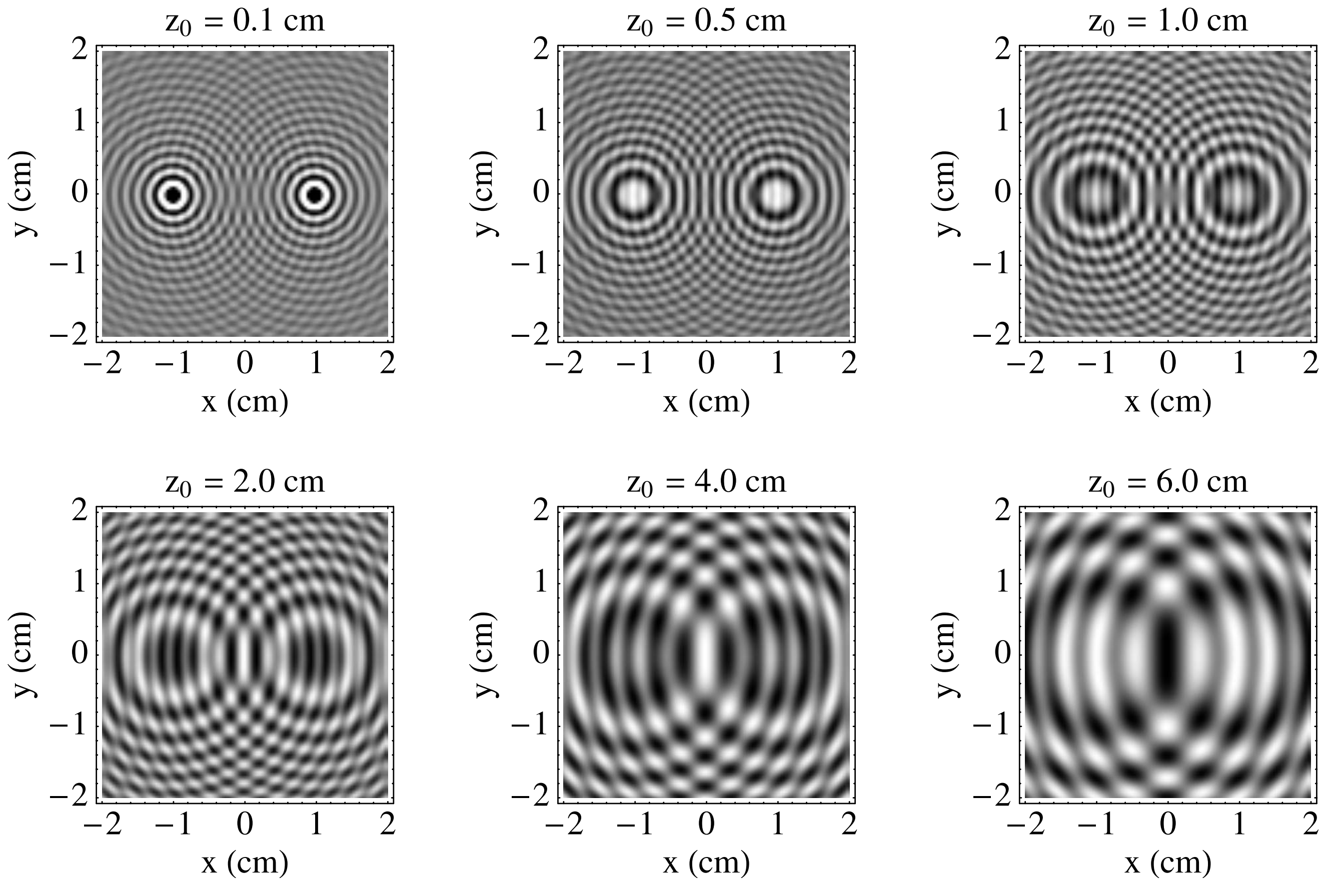

FIG. 6. Density plots of the relative pressure $|p-p_1|$ on the plane $z=0$. The depth of the absorbers is indicated for each plot. The images are displayed on a linear gray scale and each plot is normalized to its own maximum.

$$p_2(\mathbf{r}) = \frac{\alpha_0 \delta\alpha V}{D(k^2+k_2^2)} \sum_{i,j} T_{ij} G_D^{(0)}(\mathbf{r}_1,\mathbf{R}_j) \Bigg(G_1(\mathbf{r},\mathbf{R}_i) + \frac{D}{2} G_D^{(0)}(\mathbf{r},\mathbf{R}_i) - \frac{i}{8\pi}\int_0^\infty \frac{dq}{k_{2z}(q)} q T(q) J_0(q|\boldsymbol{\rho}-\boldsymbol{\rho}_1|) \times \frac{Q(q)\ell - 1}{Q(q)\ell + 1} e^{-Q(q)z_0} \Bigg), \tag{77}$$

$$p_3(\mathbf{r}) = \sum_i \delta\alpha_i V_i G_1(\mathbf{r},\mathbf{R}_i) G_D^{(0)}(\mathbf{R}_i,\mathbf{r}_1), \tag{78}$$

$$p_4(\mathbf{r}) = -\sum_{i,j,k} \delta\alpha_k V_k T_{ij} G_1(\mathbf{r},\mathbf{R}_k) G_D^{(0)}(\mathbf{R}_i,\mathbf{R}_k) G_D^{(0)}(\mathbf{r}_1,\mathbf{R}_j). \tag{79}$$

We illustrate the above results for the case of a pair of identical absorbers, as shown in Fig. 5. We take the optical properties of the medium and of the absorbers to be as in Fig. 3. The source is located at the origin and the absorbers are located on the plane $z=z_0$ with positions $\mathbf{R}_1=(1,0,z_0)$ and $\mathbf{R}_2=(-1,0,z_0)$, where all lengths are measured in centimeters. Figure 6 shows density plots of the relative pressure $|p-p_1|$ on the plane $z=0$ for different depths $z_0=0.1$, 0.5, 1.0, 2.0, 4.0, 6.0 cm. It can be seen that when the absorbers are close to the $z=0$ plane that they are well resolved and that the resolution decreases at greater depths. Note that the FWHM of the pressure along the line $y=0$ (for either absorber) is 4 mm for $z_0=1$ cm, which can be taken as a measure of the achievable resolution at that depth.

VI. CONCLUSIONS

We have considered the photoacoustic effect for multiply scattered light in an absorbing random medium as described by the diffusion approximation to the radiative transport equation. The theory was specialized to the case of one or more small absorbing inhomogeneities located in either an infinite or semi-infinite medium. Several comments on our results are necessary. First, the detection thresholds and resolution limits we have obtained must be considered to be best-case estimates. We have not directly considered the effects of systematic errors in positioning of the source and detector or other experimental parameters. Second, we have not addressed the problem of characterization of the absorbers given realistic assumptions about experimental noise and systematic errors. That is, although in principle it may be possible to detect the presence of an absorbing object, it may not always be possible to accurately estimate its position, size, and contrast. Third, when the system is not known to consist of isolated inhomogeneities, it is of interest to recover the spatial dependence of the absorption. This inverse problem has been studied by previous investigators, but without accounting for multiple scattering of the illuminating field or the influence of boundaries [14–20]. Evidently, the importance of such effects may be investigated by using the methods developed in this paper to test image reconstruction algorithms. Finally, it would be of interest to consider the photoacoustic effect in a medium where the DA breaks down. In principle, this situation could be analyzed since the necessary Green's functions for the RTE are known for both homogeneous media [21] and for collections of point absorbers [22]. These and other topics will be the subjects of future works.

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